

The longevity risk impact on the public retirement balance using the Bayesian regression -Case of Tunisia-

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Abstract

The considerable financial situation of the pension systems becomes the real problem in the pension fund, especially when we see a financial deficit in public pension systems. The financial deficit of public pension systems is the difference between the pensions of retirees (expenditures) and the contribution income of affiliated companies (revenues). The empirical impact of many variables on the pension balance is unexplained because of the complexity of the relationship between them and the permanence of the public pension system. Pension systems that have longevity generate better returns and revenues. To achieve this longevity, the pension system must make the necessary reforms by quantifying the risks. Due to the increase in demographic



risk, we focus on longevity risk in the empirical case of the Tunisian pension system, since the situation of the financial system is visibly deteriorating and reforms are urgently needed for the study. For this reason, we used Bayesian regression in R to confirm the strong relationship between longevity risk and financial equilibrium. In addition, we measured the impact of this risk on the financial equilibrium of the public pension system using Bayesian regression and Markov Chain Monte Carlo to determine the best solutions for pension system reforms.

Keywords: Public retirement, longevity risk, financial retirement balance, Bayesian regression, Markov Chain Monte Carlo (MCMC).

الملخص

يعيش الوضع المالي الكبير لأنظمة التقاعد مشكلة حقيقية في صندوق التقاعد، خاصة عندما نرى عجزًا ماليًا مستمرا في أنظمة التقاعد العمومية، وهذا العجز المالي يمثل الفرق بين النفقات العمومية على التقاعد والمتمثلة أساسا في جرايات التقاعد، وبين إيرادات الأنظمة التقاعدية، المعتمدة أساسا على مساهمات المشتركين من الأفراد والمنظمات العامة. إن الأثر التجريبي للعديد من المتغيرات على التوازن المالي للتقاعد لم تتضح معاليمه وخفاياه بعد، بسبب غموض وعدم وضوح العلاقة بينها ووبين نظام التقاعد العام. لتحقيق نظام تقاعد متوازن يمكن الاجيال القادمة من التمتع بالامتيازات التي عاشها المتقاعدين السابقين وجب البحث على الاسباب الرئيسينة التي تؤدي إلى دائمية واستمرارية هذه الأنظمة في تحقيق أهدافه بكفاءة وفاعالية. لذلك تسعى نظم التقاعد إلى إجراء الإصلاحات الضرورية والاساسية لبعض المتغيرات



كبيرا وتهديدا مباشر لمستقبل نظام التقاعد العمومي نجد زيادة طول عمر المتقاعدين وهو المتغير الذي رايناه يمثل تهدبدا ضمنيا للسلامة المالية للتقاعد وذلك بقياس مدى تأثيره على التاوازن المالي ثم تحديد مستوى خطورته، ثم تحديد تصنيفه ضمن المخاطر الأجلة فيجب الوقاية منها أو العاجلة فجب التركيز عليها لمجابهتها ضمن الأولويات الرئيسية. ولدراسة هذا المعضلة سنركز على خطر ارتقاع طول العمر المتقاعد في نظام التقاعد العمومي النوراسة هذا المعضلة سنركز على خطر ارتقاع طول العمر المتقاعد في نظام التقاعد العمومي المتقاعد ونشرك بقياس مدى تأثيره على التاوازن المالي ثم تحديد مستوى خطورته، ثم تحديد تصنيفه ضمن المخاطر الأجلة فيجب الوقاية منها أو العاجلة فجب التركيز عليها لمجابهتها ضمن الأولويات الرئيسية. ولدراسة هذا المعضلة سنركز على خطر ارتقاع طول العمر المتقاعد في نظام التقاعد العمومي التونسي ، نظرًا لأن وضع النظام المالي متدهور ويعاني من عجز مستمر ومتزايد بشكل واضح، لذلك هناك حاجة ماسة إلى إصلاحات اصلاحات وتعديلات عاجلة. لقياس هذا الخطر، استخدمنا الانحدار البايزي في R لتأكيد العلاقة القوية بين مخاطر طول العمر والتوازن المالي. بالإضافة إلى واضع المالي متدهور ويعاني من عجز المتار ومتزايد بشكل واضح، لذلك هناك حاجة ماسة إلى إصلاحات اصلاحات وتعديلات عاجلة. لقياس مائير ومتار المالي والمالي والتوازن المالي بالإضافة إلى ذلك ، قمنا بقياس العلاقة بين خطر ارتفاع طول العمر على التوازن المالي لنظام المالي. بالإضافة إلى ذلك ، قمنا بقياس العلاقة بين خطر ارتفاع طول العمر على التوازن المالي النظام المالي. بالإضافة إلى ذلك ، قمنا بقياس العلاقة بين خطر ارتفاع طول العمر على التوازن المالي التقاعد التقاعد المالي وسلسلة ماركوف مونت كارلوو لتساعدنا في تحديد ألمالي المستقبلية لإصلاحات النظام العمومي للتقاعد التونسي.

الكلمات المفتاحية: التقاعد العمومي ، مخاطر طول العمر ، التوازن المالي للتقاعد، الانحدار البايزي ، نموذج سلسلة ماركوف مونتي كارل(MCMC) .

Introduction

The functional characteristics of the pension structures are related to the monetary and social changes that improve the pension system. The symptoms of this improvement are the institutionalization of the pension system, the diversity of the structures, and the system of generosity financed and directed by the states, which can be very important. The enormous and early use of the retirement system and the generosity of the system have contributed to the deterioration of the system. Therefore, the retirement age should be suspended and contributions raised or the generous system reduced. The extensive and early use of pension provisions and the generosity of the system have contributed to the deterioration of the financial



situation of this system, which suffers from a persistent fiscal deficit. Therefore, postponing the retirement age and increasing contributions or reducing benefits have become a common element of all proposals to reform the system. This study aims at a double dimension, based on the study of the current financial situation of the public pension system and the identification of the impact of the longevity risk on this financial situation, to be able to develop appropriate solutions and implement the necessary reforms. The main question: what is the impact of the longevity risk on the financial situation of the public pension system, using the dynamic algorithm model that includes the Bayesian regression in R?

1. Public retirement system

In the literature, old age is considered negative due to insufficient gains in retirement and financial dependence. (Vaughan, E.J., Vaughan, T.M. 2001). The undertaking of rapid aging of the population destroys the pension system, Mavlutova. I. and Titova. S (2014). Jesús-Adrián Alvarez, and al, (2021), confirmed that the demographic and monetary implications of such correlations are not clear. In many countries, governments provide sufficient retirement income (Roio. Losada, A. Navarro, J. Sanchez, T. Torres (2021)). Many of researchers claim that "the threat of default is looming" (Shoag, and Daniel. (2017), for this reason, we diagnosed that retirement variables can affect the financial equilibrium. (Novy-Marx and Rauh, 2014) and (Ergungor, 2017), claim that the structures of the general public pension harm retirement. Many solutions have referred to the intervention of the authorities to prevent the system from



escalating. With the current laws, the structure of retirement has reduced the gains and durability of the system (Bachrach, 2016). In recent decades, the plans have dramatically accelerated the vulnerability of portfolios (Lu, Pritsker, Zlate, Anadu, Bohn, 2019). If a current legal obligation is financially sustainable, then intergenerational retiree equity fears that it will not be repaid (Jo. Halea, M. Bijlsmab, A. Lorenti 2021). We must explain the main types of retirement, which have been divided into two main systems represented in the distributive system and the formal system, and a third system emerged combining the two systems into one system.

The distributive system is based on the principle of solidarity between generations, which through their contributions bear the cost of salaries or capital of pensioners. The pay-as-you-go system represents a social contract between the employer, the pension provider, the employees, and the pensioners. The social contract can be changed in rules, procedures, and laws at any time by the government.

The capitalized pension system is based on capitalist thinking, i.e., trying to create wealth through saving and investment. It depends on the decision-makers in the pension organization. In general, the system converts individuals' contributions for retirement during their working lives into future savings that are used to meet their needs. Pre-arranged funds during retirement are paid in the form of a lump sum, which is paid in full upon retirement, or in the form of salary payments to the retiree, depending on the amount and duration of contributions. The contributions



saved during the working years, and then redistributed on one-time or monthly annuities.

2.1. The public retirement equilibrium.

According to (Yuying Li and Peter Forsyth, 2018), a major paradigm shift is taking place around the world, away from defined benefit pension plans to defined contribution plans. The public and private sectors are willing to take on the risks of defined benefit plans. The financial sustainability of the pay-as-you-go system is ensured by contributions that are distributed equally between generations and within each generation between different income levels and between labor and capital income. Therefore, this system achieves the goals of equity and solidarity. After retirement, the fund enters the phase of emptying and pays the retirees their financial contributions for the rest of their lives (Ian Tonks Xfi, 2006). In this context, Boado-Penas et al, 2008 warn that some policymakers and practitioners mistakenly consider the fund deficit or surplus as an indicator of financial coverage, i.e., there is confusion between the annual liquidity index and the sustainability index. According to (Daddio and Whitehouse, 2012), three main automatic mechanisms can be considered for adjusting pension values of benefit levels, of the contribution rate, of the pension amount. In general, the pension system is financially balanced if the result of the participants' contributions is succeeded by the total pension benefits. The financial equilibrium depends, on the one hand, on the number of participants and pensioners and, on the other hand, on the income of the participants and the wage mass of the pensioners. Modern



demographic forecasts show the need to look for a new source of financing for these funds. The financial equilibrium can be achieved by setting new regulatory standards to match the calculation of income with expenses and to make a continuous assessment of risks, and this is one of the rules adopted by the actuarial science of life insurance in general and pension in particular.

2.2. The Tunisian public pension situation

In the last few years, the deficit of financial situation of the CNRPS has been marked, due to the growing imbalance between the benefits paid to pensioners (expenditures) and the contributions of active workers (income) since 2005. Statistical data on the financial situation of public pensions, however, typically do not distinguish between social pooling and individual consideration, reflecting the mixed-use of the system. The analysis of the pension expenses, from 2001, has increased at a faster rate than that of its receipts. As a result, it has had a strong deficit from 2005. According to the statistical bulletins on the development of the national social security fund since 2001, the graph presents the balances of the public pension fund (CNRPS) from 2001 to 2017. Financial balances were not restored over the last decade, and the current situation has become alarming, reaching a deficit of 793,224 MD in 2017. In addition, the annual growth rate of expenditure pension has exceeded the growth rate of pension income since 2012.

According to the policy, the minimum contribution years for employees are 5 years and the contribution bases vary by 72% of the average salary. Despite this, in practice, there are many outstanding contribution cases, with the lowest



contribution rates, so the actual contribution rate is below the policy rate. Life expectancy at birth has also improved, rising from 51.1 years in 1966 to 75.1 years in 2015 and reaching around 76.9 years in 2030. Retirement years also recorded an increase reaching 19 years after retirement in 2014 and probably 19.7 years after retirement in 2029. The stability of employment and the duration of contributions mainly explain these structural characteristics of the retired population and their evolution. However, these high pension rates are not enough to provide information on the real standard of living of retirees; to have the first idea of it, it would be necessary to have information on the amount of the pensions. In 2017, the average pension was 1174 DT, or the annuity of 73% of the salary in the public sector. This is probably due to the accumulation of a long career. We notice that the average salary and the average pension have been rising continuously from 2001 until 2017. The average salary has increased from 600D in 2001 to 1600D in 2017, almost 3 times the reference salary. In addition, the average pension increased from 400D in 2001 to 1200D in 2017.

2.3. The Tunisian public retirement equilibrium

The contributions that are collected equals the pension expenses that are paid out. These contributions are considered the main source of financial investment to cover future obligations. In the case of a mass equilibrium of the pension system, the impact of longevity on retirement can only be determined at the level of the number of current pensioners after subtracting the deceased pensioners, which is the impact of the increase in the mortality rate of gross pensioners. The main



question in this paper is the following: How can we determine the impact of longevity on the financial situation of retirement?

We will introduce another balance variable, that takes into account the income and expenses for an individual account that takes into account the liabilities during the entire period of years of service and the period during all years of retirement. In a pay-as-you-go pension system with individual accounts, the Tunisian public retirement equilibrium is formulated, when the Income equals Expenses, I=D, so we have: S x A x T x ASER = P x R x ARET (1)

Where D is the expenditure, R is the number of retirees, and ARET is the number of years of retirement, I is the income, S is the average salary of the members during this period, T is the contribution rate of the members and the contribution rate of the employers, and ASER represents the years of service. In this case, we can determine the direct effect of longevity on retirement at the level of the current number of retirees after subtracting the deceased retirees and the average duration of the years of retirement in which pensions are received, which is the effect of increasing the crude mortality rate of retirees. For this reason, we must rely on the individual account balance and not forget to refer each time to the mass balance at time t to confirm the financial solvency of the Tunisian public pension system.

2. The longevity risk

Individual longevity risk (also known as individual longevity risk) refers to whether or not a person lives longer than projected. This risk could be linked to premature depletion of savings or a misallocation of investments over time



(Stallard 2006; Pitacco et al. 2009). Individual longevity risk, which can have major negative repercussions for people, poses no threat to pension system financial stability. Aggregate longevity risk, also known as trend risk, influences the entire population. It refers to the fact that the average life expectancy of a certain population will be higher than projected. In other words, it is the danger of making inaccurate predictions about future mortality rates. Total longevity risk is formed by combining specific and aggregated longevity risks (Blake Burrows 2001).

2.1. The longevity risk variable of public retirement

Longevity risk refers to the possibility that people will live longer than expected. However, pension plans are all exposed to longevity risk because they finance individuals' retirement and often promise to make payments for their lifetime. The amount needed depends on two main factors: the return on the accumulated assets and the duration of the payments. Mortality rates must also be assumed in determining the expected duration of payments, as payments are usually made until the death of the person. The degree to which pension systems are vulnerable to longevity risk depends on their structure, and in particular on the method used to calculate benefits. An individual or group who lives longer than expected is considered a longevity risk. Uncertainty about mortality rates and the potential inadequacy of pension benefits arising from underestimated life expectancies stems in large part from uncertainty about the course of mortality and the future improvement in mortality rates. Studies of population life expectancy validate that



lifespan increases over time, depending on gender. It also examines whether the mortality tables used by pension funds to assess their liabilities could potentially expose them to an expected shortfall in provisions to meet upcoming pension and annuity payments. The issue of suitability considers only a combination of level risk and trend risk: it examines the appropriateness of an index to determine if it accurately represents the underlying mortality of the covered life portfolio. Taking into account the size of a pension plan is similar to Bailey's relevance criteria. This implies that the volatility of the pension plan's liabilities relative to a specific index-based longevity swap should be significantly lower than its volatility relative to a population-based longevity swap. For starters, the system does not rely on the availability of a large set of empirical data on age-specific mortality rates, which is required for age-specific mortality models. This is especially important in areas where reliable transportation is scarce.

2.2. Tunisian longevity risk indicators.

Tunisia, like many other countries, has experienced a profound change in the pyramid structure of its population. Life expectancy has increased rapidly over the past decades and is expected to continue. An increase in life expectancy is the most interesting indication, i.e. a longer pension period. In 2016, life expectancy was estimated at 74.5 years for men and 78.1% for women, while in the early 1960s it was estimated at 41.06 years for men and 43.01 for women. This continuous increase in the retirement period leads to an additional financial burden and intergenerational income transfers in the distribution. The difference between the life



expectancy of the period 2017 and the life expectancy in 2016 shows that future improvements in life expectancies should have a slight evolution. On average, the 15 years of improvements in mortality add 3.7 years to life expectancy for men and 3.1 years for women. The number of deceased population is becoming lower and lower, which in 1960 was almost 23 per 1000 inhabitants, that is to say, 2.3 per 1000 which has become 5.5 per 1000 inhabitants, this number is very low compared to previous years. In 1985, there were 3100 public sector retirees; by 2001, the number of retirees had increased at a very strong annual growth rate to 91,822 retirees, while the number of active people increased to 541992, resulting in a rapidly decreasing dependency.

3. Bayesian regression theory.

Rdonez C, Garcia-Alvarado C, Baladandayuthapani, V (2014) have advanced Bayesian regression fashions. The authors introduce a database machine system to achieve data for whole information, after which they carry out a Bayesian regression. In addition, Ghosh J, Reiter JP, (2013) broaden Bayesian linear regression strategies primarily based totally on data from information subsets. Regression evaluation is a principal device in implemented data that targets to reply to the pervasive query of ways sure variables affect a sure outcome (structured variable). Bayesian regression seeks to version the connection among the unbiased or explanatory variables and the reaction variable via way of means of becoming a linear equation to the information. We intend to attain a first-rate version; it's miles the only version with an extraordinary explanatory predictive



power. Due to tendencies in information technology and the exponential boom of massive information in current years, statisticians have been confronted with new demanding situations inside the evaluation of massive information units.

Bayes Theory provides us with a simple rule for updating probabilities when new information appears. Bayesian theory is the key result drive of Bayesian modeling and statistics:

$$P(B_k \mid A) = \frac{P(A \mid B_k)P(B_k)}{P(A)} = \frac{P(A \mid B_k)P(B_k)}{\sum_{j=1}^{K} P(A \mid B_j)P(B_j)}$$
(2)

With S is a sample space, and B1, ..., BK, is a partition of S so that: Sk Bk = S and Bi TBj = \emptyset , and for any $1 \le k \le K$. The probability of different effects given the observed data is calculated in a distribution of possible values for the parameters., called posterior distribution. Renormalization then leads to the model weighted posterior distribution for any statistic θ :

$$2K p (\theta|y,X) = Xp (\theta|M\gamma,y,X) p (M\gamma|X,y)$$
(3)

The model prior p (M γ) has to be elicited by the researcher and should reflect prior beliefs. A popular choice is to set a uniform prior probability for each model p (M γ) \propto 1 to represent the lack of prior knowledge. The posterior value is proportional to the probability of the previous times, it is, therefore, equal to the probability x previous = C posterior x probability. Likelihood gives relative weights to all possible parameter values based on the probability that the observed value was given to each parameter value.



Subsequent simulations could use the Metropolis algorithm. Geman.S and Geman.D. (1984) talk about the optimization to locate the posterior mode in preference to simulation, and it took a little time for the Gibbs sampler to simulate the posterior distribution. therefore taking into account complete Bayesian inference. Some name the Gibbs sampler as growth in information. Metropolis and others (1953) invented the Markov Monte Carlo chain (MCMC), which simulated the dynamics of the liquid machine in equilibrium with its fuel line phase. The simplest had to simulate a Markov chain having an identical equilibrium distribution. The Markov chains via a standard direction on stochastic strategies have typically the simplest visible examples in which the kingdom area is finite or countable. Subsequent simulations may want to use the Metropolis algorithm. Geman.S and Geman.D. (1984) talks approximately the optimization to discover the posterior mode in choice to simulation. It took a little time for the Gibbs sampler to simulate the posterior distribution, therefore taking into account the complete Bayesian inference. Some name the Gibbs sampler a growth in information. Metropolis and others (1953) invented the Markov Monte Carlo chain (MCMC), which simulated the dynamics of the liquid system in equilibrium with its gas line phase. They handiest needed to manufacture a Markov chain having the same equilibrium distribution. The Markov chains thru a fashionable path on stochastic techniques generally have the handiest seen examples wherein the dominion vicinity is finite or countable. If the dominion vicinity is finite and written, then the initial distribution can be associated with a vector $\lambda = (\lambda 1, ..., \lambda n)$ described in via manner way of:



$$Pr(X1 = xi) = \lambda i$$
 (4)

With, i = 1, ..., n. The transition probabilities can be associated with a matrix P having elements pij defined by:

$$Pr(Xn+1 = xj | Xn = xi) = p \quad (5)$$

When the state area is infinitely countable, we can think about a vector and an infinite matrix. However most of the Markov chains of interest in MCMC have unnumbered state space, then we tend to cannot consider the initial distribution as a vector or the transition chance distribution as a matrix. We have to think of them as an unconditional probability distribution and a conditional probability distribution. We will use multiple R diagnostic plots and influence statistics to diagnose however well our model works the data. We will eliminate an outlier to ascertain how it changes the fit of the model. These diagnoses embrace Residues vs adjusted values.

4. The results

The paper intends to take a look at and implement Bayesian regression models on different datasets. The aim of Bayesian linear regression is not continually to discover the quality and precise fee of the model parameters, however, is a substitute to determine the posterior distribution of the model parameters. This provides a Bayesian assessment for comparisons with beyond distributions. We consider Y (Public retirement balance) as a dependent variable and X



(especially the longevity risk) the independent variable, usually, the question of interest is how we are able to assume Y based mostly on X?

:So we can make as hypothesis as following

 H_0 : There is no relationship between the public retirement and the longevity risk, in the level of signification 0.05.

 H_1 : There is a relationship between the public retirement and the longevity risk, in the level of signification 0.05.

The dataset describes the financial equilibrium of the overall public pension machine in Tunisia, as well as several precise elements, which consist of durability, and earnings growth... The records include 18 variables; which means we can have 262144 combos of models.

In our case, our whole model will embody 18 variables that are probably critical in predicting the financial balance of public retirement as indicated in table 1:

| Variables | Descriptions |
|-------------------|--|
| Longevity | This variable considers mortality rates and years of retirement. |
| Retirement years | The years of survival after retirement. |
| Salary Increasing | After the agreement between the UGTT and the public organization. |
| Annuity rate | The pension liquidation rate is applied on the reference salary. |
| Pensions | A financial reward upon retirement after a period of contribution. |
| Expenses | The overall amount of pensions paid to retirees. |

Table 1: The variables of the Bayesian regression model



Retirees number Retirees deaths New retirees New affiliates Affiliates Deaths Affiliates number Salary average Employees Contribution rate Service Years Cash flows Number of retirees receiving pensions during survival after retirement. Number of retirees who died after retirement in year t. Number of new retirees members in year t. Number of new retirees receiving pensions in year t. Number of affiliates who died in activity in year t. General number of affiliates at the end of the year t The average of s wages served to the affiliates during the year t. General number of affiliates at the end of the year t The sum of the affiliate and the employer contribution rate. The average number of years of affiliate careers at time t. The total amount of contributions paid into the fund

For robustness take a look at primarily based totally on eligibility for CNRPS records reviews from 2001 to 2017, I did many calculations to get the very last records. Table 2 suggests precis information for the running records. Columns 2 to six offer a range of observations, minimum, maximum, and well-known deviation. We must notice that we have 6 missed observations of sturdiness variables.

Table 2. Descriptive statics

| Variables | Observations | Minimum | Maximum | Mean | Std. Deviation |
|------------------|--------------|---------|---------|----------|----------------|
| Longevity | 11 | 65 | 66 | 65.27273 | 0.467099 |
| Retirement-years | 17 | 8 | 10 | 9.35294 | 0.606339 |
| Salary added | 17 | 19 | 70 | 41.52941 | 19.33623 |
| Annuity-rate | 17 | 0.654 | 0.7283 | 0.700312 | 0.024427 |
| Pensions | 17 | 351 | 1174 | 701.4118 | 250.4955 |
| Expenses | 17 | 592 | 4275 | 1802.294 | 1108.757 |



| Retirees-number | 17 | 96218 | 246011 | 159703 | 47584.45 |
|---------------------|----|---------|--------|----------|----------|
| Death-retirees | 17 | 2096 | 4580 | 3089.059 | 765.209 |
| New-retirees | 17 | 6906 | 20737 | 13267.41 | 4001.158 |
| New-affiliates | 17 | 20873 | 47243 | 30690.06 | 8371.519 |
| Affiliates-number | 17 | 541992 | 782304 | 655735.2 | 82895.5 |
| Salary-average | 17 | 0.62687 | 1.7917 | 1.11765 | 0.332106 |
| Employees-number | 17 | 541992 | 782304 | 655735.2 | 82895.5 |
| Contribution-number | 17 | 0.142 | 0.207 | 0.179059 | 0.028453 |
| Service-years | 17 | 28 | 32 | 30 | 1.224745 |
| Financial balance | 17 | -793 | 79 | -127.588 | 208.6999 |

The estimation model is very complex and it is based on different techniques such as likelihood, general estimation equations, or Bayesian methods. Bayesian models offer, in some cases, the only way to obtain a reasonable estimate of model parameters, due to the possibility of including prior knowledge of these parameters.

4.1. Estimated coefficient and Residual analysis.

The perfect used model in this search is the Bayesian regression, to build a good predictive model. We can see that the residuals show a distribution close to normal with a straight bias with a median of -2.658e-14. When evaluating the adequacy of the model with the data, we must look for asymmetrical distribution between these points on the mean value of zero (0). In our example, we can see that the distribution of the residuals does not seem strongly symmetrical.

Table 3. The data frame of the Residuals



| Min | 1Q | Median | 3Q | Max |
|-----------|-----------|-----------|----------|----------|
| -1.90E-12 | -1.65E-13 | -2.66E-14 | 1.32E-13 | 1.48E-12 |

The best way to obtain Bayesian estimates of parameters is the sample from the posterior with a Markov chain Monte Carlo. Theoretically, if we wanted to predict the impact of longevity on the financial equilibrium of public retirement, we would produce estimates of the coefficients, and then use them in the formula of the model. The dependent variable is essentially the financial equilibrium of public retirement; it was only 2.823E-11. There is an inverse relationship between longevity and the financial balance, which indicates a result of -4.039E-13 as in table 4 below.

| | Estimate | Std.Error | t value | Pr(>/t/) |
|------------------|-----------|-----------|-----------|-------------|
| (Intercept) | 2.82E-11 | 1.05E-11 | 2.70E+00 | 0.00846 ** |
| Longevity | -4.04E-13 | 1.19E-13 | -3.41E+00 | 0.00103 ** |
| Retirement-years | 2.52E-14 | 1.14E-14 | 2.22E+00 | 0.02941 * |
| Salary-added | -5.13E-13 | 1.70E-13 | -3.03E+00 | 0.00330 ** |
| Annuity-rate | -3.64E-13 | 1.24E-13 | -2.93E+00 | 0.00445 ** |
| Pensions | 7.71E-15 | 2.42E-15 | 3.18E+00 | 0.00208 ** |
| Expenses | -1.00E+00 | 1.51E-16 | -6.63E+15 | < 2e-16 *** |
| Retirees-number | -2.16E-12 | 1.42E-12 | -1.52E+00 | 0.133 |
| Death-retirees | -2.16E-12 | 1.42E-12 | -1.52E+00 | 0.133 |
| New-retirees | 2.16E-12 | 1.42E-12 | 1.52E+00 | 0.13299 |
| | 1 | | | |

Table 4. Coefficients estimated



| New-affiliates | -6.22E-14 | 1.27E-13 | -4.92E-01 | 0.62431 |
|---------------------|-----------|----------|-----------|------------|
| Death-affiliates | 6.23E-14 | 1.27E-13 | 4.93E-01 | 0.6236 |
| Affiliates -number | -6.22E-14 | 1.27E-13 | -4.92E-01 | 0.62432 |
| Salary-average | -1.31E-15 | 1.68E-15 | -7.80E-01 | 0.43774 |
| Employees-number | 6.22E-14 | 1.27E-13 | 4.92E-01 | 0.62432 |
| Contribution-number | -2.62E-12 | 9.59E-12 | -2.73E-01 | 0.7857 |
| Service-years | -4.81E-15 | 6.62E-14 | -7.30E-02 | 0.94223 |
| Income | 1.00E+00 | 4.49E-15 | 2.23E+14 | <2e-16 *** |

In general, a p-value of 5% or less is a good sign for this model. The null hypothesis: H0: $\beta 1= 0$, which means that the hypothesis of no relationship between longevity and financial balance was rejected. Alternatively, the hypothesis: H1: $\beta 1 \neq 0$, which concludes some relationship between the longevity and the financial balance. The significance level of the p-value is less than 0.05, in our case the value p, concerning longevity is only 0.00103, which shows some relationship between equilibrium and longevity. Therefore, we reject the null hypothesis and we conclude the hypothesis: H1: $\beta_1 \neq 0$ that it exists some significant relationship between the longevity and financial balance.

In general, t values are also used to calculate p values, the t-test statistic determines the correlation between the response and the predictor variables. We can use the t-test to find that the p-value that corresponds to the t-value of -3.41 is 0.00103. since this p-value is less than 0.05, so we confirm that it exists some relationship between the longevity risk and the financial balance.



The squared statistic R2 provides a measure of how well the model fits the actual data. It takes the form of a proportion of variance. R2 is a measure of the relationship between the predictor variable (financial balance) and the response (longevity). In our search, the R2 is one. On the other hand, about 100% of the response variable (financial balance) is explained by the predictor variable (longevity). The three stars of "signify. Codes" represent a very significant relation. The most significant variables are the cash flows and the expenses, which directly affect the financial balance of the Tunisian public retirement. Then we find the other signs of two stars like longevity, pension, salary increase, and pension annuity rate. Finally, we find a single star; that means the variable has a weak effect on the retirement equilibrium, like the years of retirement.

The residuals are essentially the difference between the observed real response values (the financial balance or equilibrium) and the prediction values by the model. The expected vs actual values after fitting regression in R plots predicted values versus actual values contain two axes: the X-axis shows the model's predicted values under BMA, while the y-axis shows the data set's residual values. In the Residual chart, the random point in the red line should be flat. It should not increase or decrease as the fitted values increase. this condition needs to be avoided, especially for the P-value is greater than the significance level. It means the residuals should be zero.



Graph 1. The residuals charts.

The top left residual vs fitted plot indicates that the random points in the red line are not flat, they increased slightly and they decreased slightly. It is not grave, because we had a p-value smaller than the significant level of 0.05 it was only 0.00103. The top right normal Q-Q plot is to make sure that the residual follows a normal distribution. We have a good residuals distribution because the random points fall perfectly on the diagonal line, it is clear that the residuals were normally distributed. The scale location plot on the bottom left is used to measure the square root of the standardized residuals against the fitted value. Therefore, for the points on the regression line, the value of y should be close to zero. The plot indicated



that the line regression is flat in one and is not too close to zero. On the bottom, is a right plot of standardized residuals against the advantage that measures how much each data point influences the regression.?

After running the full model with all the variables involved, we got an adjusted R² of 1, this is an optimal fitted R², which means that we cannot improve the model. The posterior probability that longevity is included in the model is 0.09277115. Furthermore, the most probable model, which has a posterior probability of 0.2923, includes an interception, years of retirement, salary increase, annuity rate, pension, expenses, number of retirees and members...

4.2. Performing the models

One of the main advantages of Bayesian methods is that posterior inference is quite straightforward using MCMC output since the methods provide direct samples of parameters of interest. We can also provide 95% credible intervals for these coefficients.

| | 2.50 % | 97.50 % | Beta |
|------------------|---------------|------------------|------------|
| (Intercept) | -1.31E+03 | <i>-1.31E+03</i> | -1.31E+03 |
| Longevity | -6.71E-06 | 9.03E-06 | 9.86E-04 |
| Retirement-years | -4.83E-07 | 4.54E-07 | -1.62E-13 |
| Salary added | -6.70E-06 | 7.53E-06 | 4.63E-12 |
| Annuity-rate | -2.20E-06 | <i>3.51E-06</i> | 2.43E-06 |
| Pensions | -9.63E-08 | 1.08E-07 | -8.10E-06 |
| Expenses | -1.000000e+00 | -1.00E+00 | -1.9999844 |

Table 5. The indicator parameters of the Bayesian regression model.



| Retirees-number | -5.41E-10 | 1.05E-09 | -1.85E-06 |
|---------------------|------------------|--------------|------------------|
| Death-retirees | 0.00E+00 | 0.00E+00 | 8.06E-07 |
| New-retirees | 0.00E+00 | 0.00E+00 | -1.14E-09 |
| New-affiliates | -5.95E-10 | 6.86E-10 | <i>-3.14E-09</i> |
| Death-affiliates | <i>-1.02E-08</i> | 2.33E-08 | 5.46E-13 |
| Affiliates number | <i>-3.79E-11</i> | 3.58E-11 | 5.24E-10 |
| Salary-average | -5.22E-08 | 6.16E-08 | <i>9.40E-06</i> |
| Employees-number | -9.43E-11 | 7.20E-11 | <i>4.07E-09</i> |
| Contribution-number | -2.15E-04 | 1.829465e-04 | 1.45E-02 |
| Service-years | -1.95E-06 | 1.91E-06 | 7.64E-05 |
| Income | 1.00E+00 | 1.00E+00 | 1.00E+00 |

By comparing the coefficients of the best model with those of the full model, we see that the 95% credible interval for the longevity variable is the same. We believe there is a 95% chance that the longevity score increases from 0.0000903 to 0.000986. The expenses have a larger effect where we believe that there is a 95% chance to increase from -1 to -2.

4.3. Robustness and prediction of the models using MCMC.

A posterior probability, in Bayesian regression, is the updated probability of the prior probability calculated. The posterior mean of the Intercept $\beta 0$ is now the sample mean of the response variable *Y* financial balance. We use the subset argument to plot only the coefficients of the predictors. The color of each column is proportional to the logarithm of the log posterior probability. Patterns of the

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same colors have similar posterior probabilities. In our case, we have 14 models as mentioned in the X-axis of the possible models in the graph below.



Graph 2. Log posterior probability.

The red variables in this plot are essentially the revenues, expenses, and pensions, which directly affect the financial balance of the Tunisian public pension. The excluded variables from a model are shown in black for each column, while included variables are colored, the color is related to the logarithm of the posterior probability. The logarithms of the posterior probabilities are scaled so that the 0 corresponds to the lowest probability model among the first ones so that the values on the axis correspond to the log Bayes factors to compare each model to the other model.



5. Conclusion

We have concluded in this paper that the impact of longevity risk on the financial equilibrium of public pensions is slightly negative using the Bayesian regression model. The simulations developed to lead to the following main conclusion: the pension system, as currently defined, has been affected by demographic structural change, especially longevity risk, but it will become more significant in the coming years. Given the country's specific social, economic, and demographic parameters, the Tunisian authorities have envisaged several types of parametric and structural reforms to reduce these losses. As we can see, these reforms have not changed the financial situation of the pension fund.

The pension fund cannot reach financial equilibrium in the case of longevity risk and the generosity of the pension system. Moreover, the adjustment of contributions proposed by the government and experts may lead to a severe economic and social crisis. If the legislation remains unchanged, this financial situation will require the implementation of certain reform measures, otherwise, it will become even more dangerous in the medium and long term and the public pension system will no longer be able to pay the pensions of future retirees.

This financial imbalance, which continues to grow year by year, requires urgent government intervention to reduce budgetary losses and ensure the sustainability of the plans. Maintaining the same parameter values (replacement rate, contribution duration, contribution rate ...) will inevitably lead to a budget loss.



We propose such parametric reforms, first, increasing the collected contributions with a decrease in the pension level, increasing the retirement age, decreasing the pension rate, and decreasing the reference salary and pension rate. We have another proposal for structural reform by introducing a second capitalist pension system leading to a mixed pay-as-you-go pension system.

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